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## A MARKET DYNAMICS MODEL FOR NEW INDUSTRIAL PRODUCTS AND ITS APPLICATION

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New product planning models attempt to predict the market consequences of product line and product design decisions. One output of such models, especially those driven by subjective or market research data, is usually theoretical market shares based upon consumer preferences under idealized conditions. This paper describes a class of models that bridge the gap between such theoretical market shares and dynamic sales forecasts. This model accounts for differences in customer awareness of different products, for differences in product announcement dates, for differences in product availability, for differences in marketing efforts, for customer inertia and for customer purchasing delays. The paper describes specific details of such a model used in a system developed for market analysis of high speed nonimpact computer printers.

**(New Industrial Products; Stochastic Choice Models; Market Dynamics; Product Planning)**

### 1. Introduction

New product planning models attempt to predict the market consequences of product line and product design decisions. They are used to aid decision makers who must define new product designs, decide whether to finish developing and launch proposed new products, and make pre-launch pricing and marketing strategy decisions. These models are particularly important in industrial and durable product markets where test marketing is economically infeasible and such decisions must rely on market research-based sales forecasts. Several such models are discussed by Choffrey and Lilien (1980) and in a recent survey by Shocker and Srinivasan (1979). Hauser and Urban (1977) describe such a model and its use in designing a health insurance plan. In a paper closely related to this one, Oren, Rothkopf and Smallwood (1980) describe the development, use and impact of such a model of the market for high speed nonimpact (e.g., scanning laser) computer printers. Models of this type may be supported by various combinations of market information, market research data and subjective assessments. Early in the development of a truly new product, only subjective estimates are likely to be available. Later, market research may be conducted to support the development effort and the go/no go decision. When the product is already launched or if prior experience with similar products is available, sales data may be a key input. One output of such models, especially those driven by subjective or market research based data, is usually theoretical market shares—sometimes called “potentials.” These are informative outputs in their own right, but it can be a long way from these “potentials” to a sales forecast over time that can be used to support a go/no go product design or pricing strategy decision. Currently

available models that produce dynamic forecasts for new products are aimed at frequently purchased products and are designed to use test market data (for a recent survey of such models, see Narasimhan and Sen 1983). These types of models, however, have not been used for expensive industrial products where marketing decisions must be made long before production and test marketing are feasible. Both Shocker and Srinivasan's survey (1979) and a recent paper by Berkowitz and Haines (1982) comment on the need for ways to bridge the gap between theoretical market shares and dynamic forecasts. This paper discusses market dynamics models that bridge that gap. Specific details of such a model which was used at Xerox in the work reported by Oren, Rothkopf and Smallwood (1980) are described in the context of the market for high speed nonimpact computer printers. The approach, however, is quite general and can be particularly useful in analyzing markets for new industrial or durable products.

The choice probabilities or market shares that provide the starting point for the market dynamics models we describe are typically outputs of some form of value/choice model. These models are usually designed to answer the following question: Given full awareness and equal availability (marketing strength, etc.) of products with given characteristics, what fraction of a representative subpopulation would prefer each product? Such data may be provided for various points in time. In converting such potentials into a forecast, a market dynamics model must account for differences in customer awareness of product existence and characteristics. It must account for the differences in the announcement dates, availability and marketing effort supporting the products and for the effect of these factors on lags in preference realization. It must also account for market inertia. This involves having future preferences depend upon current purchases. In addition, the market dynamics model in the computer printing forecasting system also had to account for some projected changes in the composition of the population of potential customers and to forecast product usage as well as placements.

Within Xerox, there has long been recognition of the need for calculations that bridge the gap between potentials and forecasts by accounting for the factors listed above. Until the computer printing forecasting system was built, however, there were no formal models for making these calculations. The calculations that were done, while often lengthy and detailed, were somewhat ad hoc and involved arbitrary "smoothing" steps. This arbitrariness reduced the value of comparisons of forecasts under alternative assumptions. Even though the calculations were closely "audited," the arbitrariness tended to undercut the credibility of the forecasts. This credibility was important since the forecasts were usually prepared by the proponents of controversial product development programs. Furthermore, because the calculations were lengthy and had to be audited, they tended to delay the preparation of forecasts. Thus, accuracy, credibility and speedy results were important design objectives of the market dynamics model in the computer printing forecasting system. Speed was achieved by systematizing the above process in a computer model that incorporated all of the important factors and eliminated the need for many approximations. While these approximations had seemed reasonable, their effects were hard to evaluate. Credibility, as well as speed in the auditing process, was achieved by carefully segregating and simplifying the necessarily subjective inputs that were used in the calculations.

This paper is organized in the following way: In order to introduce some notation and so that this paper can be read independently of the earlier paper by Oren, Rothkopf and Smallwood (1980), we first present a brief overview of Xerox's computer printer forecasting system. Next, we describe the dynamics model in that system. For ease both of explanation and of generalization, this description is done in two phases. §3 gives a simplified version of the model, and §4 explains the treatment of various

complicating factors. Next, §5 discusses the use of the model. §6 addresses issues related to model evaluation. §7 discusses both this dynamics model and various kinds of variations and generalizations of it. An axiomatic development of certain characteristics of the dynamics model is relegated to the Appendix.

### 2. The Computer Printing Forecasting System

The computer printing forecasting system uses a calibrated, market research-based value/choice model along with subjectively supplied new product and economic forecast data to generate potentials for each product as a function of time. Figure 1 shows the basic components of the forecasting system and, in general terms, the input and output information of each. The value/choice model is completely disaggregated; in effect, a forecast is generated for the subpopulation of potential customers (computer centers) represented by each of the 92 market research respondents. These separate forecasts are then aggregated to produce the total market forecast. The respondents were randomly sampled from a total universe of 5000 targeted computer sites with equipment value exceeding one million dollars. The sample was divided into twelve cells which were defined according to three criteria: the type of main frame computer possessed by the computer center (IBM 360, IBM 370, non-IBM), the size of the center measured in terms of equipment value (1-4M\$, ≥ 4M\$) and the number of existing high speed printers (1-3, ≥ 4). The sampling strategy and cell definitions were decided upon by the team members who represented the market research department at Xerox.

All respondents in a cell had the same sampling probability and thus were given equal weights, i.e., they were assumed to represent equal subpopulations. The total population represented by each cell varied, however. Furthermore, since the sample was randomly drawn from the entire universe, the cells were disproportionately represented in the sample. This was accounted for by computing the weight of each respondent, after the sampling was complete, based on the known cell population and the number of samples that fell in each cell. The resulting subpopulations represented by each respondent ranged from 17 to 174 sites. As a result, the sampling error within some cells was considerably higher than the design criteria used to determine the sample size. We, therefore, expected to produce credible projections only for the entire market and not for individual cells.

For each respondent, the value model part of the value/choice model consists of

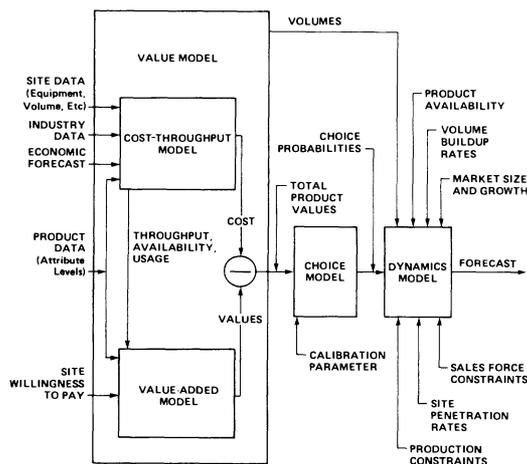


FIGURE 1. Overview of the Xerox Computer Printing Forecasting System.

two segments: A detailed cost/throughput model of the economics of computer printing at that respondent's computer center, and a model of that respondent's willingness to pay for various value added factors not included in the economic model. A telephone interview and at least two site visits with preliminary model runs were used to develop a reliable version of the cost/throughput model. The completed cost throughput model gave a detailed picture of the computer center's potential usage of a variety of potential nonimpact computer printing products. In particular, for each type of nonimpact printer it gave the nominal number of printers that would be required and identified and characterized the present printing volume that could be and that would be assigned to those printers.

The factors considered in the value added segment of the value model included throughput differences, print quality differences, differences in brand preference, reliability differences, on-line versus off-line preference, and reluctance to change from impact to nonimpact printing. These values were elicited during the site visits using a dollar metric approach. Where printers are offered with different configurations or different price plans, the combined value model (i.e., the cost/throughput plus value added model) selects the preferred alternative and uses its value.

The choice model converts the set of values assigned to the different products by a market research respondent into a set of choice probabilities for the subpopulation represented by that respondent. These choice probabilities for each respondent under the market conditions projected for each forecast year form the key input to the dynamics model. By assuming that the measured values contain additive unbiased and independent random noise having a Gumbel (1958) extreme value distribution, it can be shown that the choice probabilities will follow the well-known multinomial logit distribution (see McFadden 1974, Oren 1974). This noise may be attributed to heterogeneity of the subpopulation, preference inconsistencies and measurement errors.

The parameter in the Gumbel distribution that controls the noise's variance was used as a calibration parameter adjusted (by trial and error) so that the computer aggregated potentials matched the aggregated potentials produced by standard concept tests. These tests were administered to the same respondents for two base case sets of competing products. In this calibration process, we also corrected for systematic errors in the value added model due to omissions revealed by respondents' open-ended comments.

At this point, it is convenient to introduce some formal notation both to make some of the above statements precise and for our subsequent description of the dynamics model. Let  $V_i^k(t)$  denote the value assigned to product  $i$  in a subpopulation  $k$ , and let  $P_i^k(t)$  represent the probability that a potential customer of type  $k$  will prefer product  $i$  at time  $t$  given the choice set  $Z(t)$ . According to the multinomial logit model,

$$P_i^k(t) = \exp(rV_i^k) / \sum_{j \in Z(t)} \exp(rV_j^k) \quad \text{for } i \in Z(t),$$

$$= 0 \quad \text{for } i \notin Z(t).$$

The parameter  $r$  is the calibration parameter referred to above, and it is related to the standard deviation,  $\sigma$ , of the underlying Gumbel noise distribution through the relation  $r = \pi / (\sqrt{6} \sigma)$ .

If  $W^k(t)$  is the number of potential customers in the subpopulation represented by respondent  $k$  and if  $n_i^k(t)$  is the nominal number of computer printers that respondent  $k$  would purchase if he selects concept  $i$ , then the total gross potential for product  $i$  at time  $t$  is given by  $\sum_k W_i^k(t) n_i^k(t) P_i^k(t)$ . Such gross potentials are a key output of the value/choice model. These, however, are static predictions that do not take into

consideration, for instance, the effects of market inertia and various decision variables affecting product availability (e.g., introduction dates, production rates and sales force size). The primary function of the dynamics model is to capture such effects in order to account for the discrepancy between the static gross potentials and a forecast of installed printer populations.

Several of the factors that were included in the dynamics model of the computer printing forecasting system introduce complications that tend to obscure the simple basic logic underlying the dynamics model. Furthermore, some of these complications are easier to understand once that basic logic is understood. Therefore, we first present a basic version of the dynamics model that contains the core of the model used in the computer printing forecasting system. This part of the model contains its most “portable” features—those which we believe have general applicability and which were actually employed in other forecasting systems for different products. Then the subsequent sections describe the treatment of a variety of complications that are primarily relevant to the computer printing application.

### 3. The Basic Form of the Dynamics Model

The basic dynamics model captures the effect of inertia and lags in preference realization. It also accounts for differences in product availability due to different introduction dates and different market penetration rates. In principle, this dynamics model consists of parallel nonstationary Markovian population processes corresponding to the different subpopulations. For simplicity, we describe the model for a single subpopulation and suppress the superscript  $k$  denoting the subpopulation.

The structure of this dynamics model is based upon the distinction between preference and possession of a product. Thus, the differences between product potentials and product placements are attributed to lags between changes in the preference for products and in their possession. These lags are due primarily to availability and penetration rate factors.

The states of the Markovian population process are characterized by the pairs of preferred and possessed product. State  $\{j|i\}$ , for example, reads “has product  $i$ , prefers product  $j$ .” The transitions between these states are of two types: “preference transitions” representing changes in preference and “possession transitions” representing actual product acquisitions. Figure 2 illustrates a typical state transition diagram

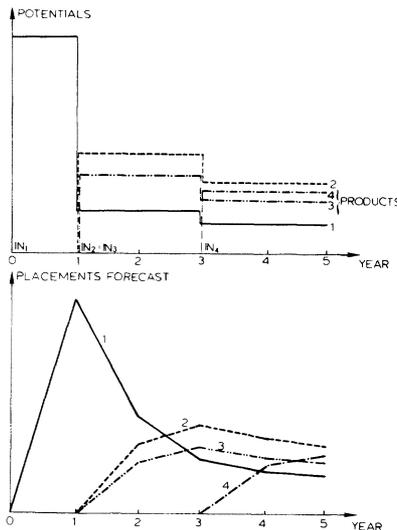


FIGURE 2. State Space and Sample Transitions for the Basic Model.

for this model. In it, states are arrayed by the possessed and preferred product. The horizontal transitions in that diagram represent preference changes while the vertical transitions are possession changes. The double arrows in Figure 2 illustrate a sample realization of the state transitions by a particular customer. He starts by having no product and preferring product  $A$ . Then, upon acquisition of product  $A$  he moves to state  $\{A|A\}$  (has  $A$  wants  $A$ ). Subsequently, due to the introduction of product  $B$ , a price reduction or other reason, he changes his preference from product  $A$  to  $B$  while he still owns product  $A$ . Hence, his new state is  $\{B|A\}$  (has  $A$  wants  $B$ ). Finally he replaces his current product  $A$  with his preferred product  $B$ , thus moving to state  $\{B|B\}$  (has  $B$  wants  $B$ ). The transition probabilities for the preference transitions are derived from the choice probabilities produced by the value/choice model. The possession transition probabilities, on the other hand, are obtained from subjective assessments based on historical sales data, judgement and marketing plans (one typically performs a sensitivity analysis with respect to such estimates).

We assume that immediately after an acquisition a customer prefers what he has just acquired. Hence, all of the possession transitions are to states on the diagonal of the state diagram  $\{i|i\}$ . In this model, we also assume that all of the possession transitions are vertical. This later assumption may be easily relaxed. Indeed, in subsequent applications we allowed “bleed transitions” representing acquisitions of a product by customers who did not obtain their first preference due to its low penetration rate. Such bleed transitions could represent customer unawareness of their preferred products or a conscious willingness to settle for second or lower choice products due to difficulty in obtaining their first choice. Cancellations and acquisitions by new customers are handled in this framework by introducing a “null” product having zero inertia and instantaneous penetration.

We now formalize the above description in mathematical terms. First, to account for inertia we recalculate the choice probabilities for each product conditional on the presently possessed product. This gives for each time  $t$  and subpopulation  $k$  a preference matrix whose  $i, j$  component  $P_{i|j}^k(t)$  is the probability that a random customer in subpopulation  $k$  prefers product  $i$  at time  $t$  given that he possesses product  $j$ . Let  $I_{i|j}^k(t)$  be the “inertia” (i.e., the disutility) of switching from product  $j$  to  $i$  at time  $t$ , ( $I_{i|i}^k(t) = 0$ ). Such values have been obtained from respondents along with other values employed in the value/choice model. Again suppressing the superscripts,  $P_{i|j}(t)$  is given by the logit model as

$$\begin{aligned}
 P_{i|j}(t) &= \exp[r(V_i(t) - I_{i|j}(t))] / \sum_{l \in Z(t)} \exp[r(V_l(t) - I_{l|j}(t))] && \text{for } i \in Z(t), \\
 &= 0 && \text{for } i \notin Z(t).
 \end{aligned}$$

A convenient way to calculate the preference probabilities  $P_{i|j}(t)$  for a partial set of available products is to obtain first the preference matrix  $[P_{i|j}^*(t)]$  assuming all products are available. The preference matrix for the case in which only a product subset is available can be obtained by setting the columns corresponding to the unavailable product to zero and renormalizing each row so that it sums to unity. This approach is particularly useful when the values  $V_i(t)$  and inertias  $I_{i|j}(t)$  are not time dependent so that the probabilities  $P_{i|j}^*(t)$  are time invariant and need to be calculated only once.

The preference transition probabilities are calculated from the changes in the preference probabilities as shown below. Let  $\phi_{i|l|j}(t)$  denote the transition probability from state  $\{i|j\}$  to  $\{l|j\}$  and let  $\Delta P_{i|j}(t+1) = P_{i|j}(t+1) - P_{i|j}(t)$ . Then we have

four distinct cases as follows. For  $l = i$ ,

$$\phi_{ii|j}(t+1) = \begin{cases} 1 + \frac{\Delta P_{i|j}(t+1)}{P_{i|j}(t)} & \text{if } \Delta P_{i|j}(t+1) < 0, \\ 1 & \text{otherwise.} \end{cases}$$

In other words, a state whose preference probability decreases loses a fraction of its population which equals the proportional change in its preference probability. Otherwise, it retains its entire population. For  $l \neq i$

$$\phi_{il|j}(t+1) = \begin{cases} 0 & \text{if } \Delta P_{i|j}(t+1) > 0 \text{ or } \Delta P_{l|j}(t+1) < 0, \\ \frac{|\Delta P_{i|j}(t+1)|}{P_{i|j}(t)} \cdot \frac{2\Delta P_{l|j}(t+1)}{\sum_m |\Delta P_{m|j}(t+1)|} & \text{otherwise.} \end{cases}$$

Namely, there are no transitions out of a state whose preference probability increases or into a state whose preference probability decreases. Furthermore, the transitions from losing states to gaining states are proportional to their respective changes in preference probabilities. (Note that the quantity  $\frac{1}{2} \sum |\Delta P_{m|j}(t+1)|$  equals the sum of all increases in preference probabilities since the sum of such increases equals the sum of the corresponding decreases.) Appendix A contains a more compact representation of the formula for  $\phi_{il|j}(t)$ , described above, and shows how this formula can be derived from the following two axioms:

1. If the preference probability of a product decreases, then the customers preferring it are all ones that preferred it in the previous year. Likewise, if the preference probability of a product increases, all of the customers who preferred it in the previous year will continue to prefer it.
2. Given that a customer changes his preference, his new preference is independent of his previous ones.

It should be stressed here that while these axioms produce plausible results they have not been validated empirically and are stated merely to highlight the logic of the proposed model. While the first axiom is quite reasonable, the plausibility of the second axiom would depend on the definition of "distinct products." If products were defined such that some share more similarities than others, then prior choices could provide information on subsequent choices and the axiom would be improper. Such similarities among subsets of products, however, would also cause problems in the logit models which generate our static choice probabilities. Product independence must, therefore, be a key provision for the validity of both the logit model and Axiom 2 above. This independence condition can be satisfied at the product definition level by grouping similar products and representing each such group by a single representative product. Choices among variants of the same representative product can be handled by a second level choice model.

The calculation indicated by the formula shown above can be explained as a redistribution of the expected population among the states sharing the same possession. For products whose preference probability decreased, the expected population in each state involving preference for that product is decreased in proportion to its probability—i.e., if the preference probability of a product is cut in half, half of the expected number of customers in each state preferring that product are lost.

The expected number of customers leaving states are totaled and then distributed among the states whose preference probability increased in proportion to the magnitude of the increases. Consider, for example, a four-product case in which the population in each preference state of customers possessing product  $j$  is  $N_{i|j}$ ,  $i = 1, 2, 3$ ,

4 and the preference probabilities  $P_{i|j}$  declined for products 1 and 2 and increased for products 3 and 4. Omitting the subscript  $j$ , the expected population changes due to preference transitions,  $\Delta N_i$ , are given by

$$\begin{aligned}\Delta N_i &= N_i \Delta P_i / P_i && \text{for } i = 1, 2, \\ \Delta N_i &= (\Delta N_1 + \Delta N_2) \Delta P_i / (P_3 + P_4) && \text{for } i = 3, 4.\end{aligned}$$

These operations, which account only for the preference transitions, are performed separately for each group of states sharing the same possessed product. It can be easily verified that if the preference distribution of customers possessing a specific product was proportional to the corresponding choice probability distribution, i.e.,  $N_{i|j} = N_j \cdot P_{i|j}$ , then the above calculations would simply imply  $\Delta N_{i|j} = N_j \cdot \Delta P_{i|j}$ . Typically, however, the preference distributions are not proportional to the choice probability distributions due to the differential penetration rates of the different products which cause disproportional depletion of their preference pools. Important features of this model which distinguish it from the various models surveyed by Narasimhan and Sen (1983) are its systematic treatment of differential penetration rates and capability to track the resulting "misalignments" between preference distributions and choice probabilities.

It should be further noted that the above procedure secures a pool of potential customers to any newly introduced product even if it is inferior to its predecessor. This is because upon its introduction a product's preference probability increases from zero to the value determined by the logit model. The relative strength of the new product, however, determines the size of the potential customers pool for the new product. By the same logic, once a product is on the market its potential and actual customer pools will be eroded with every new product introduction unless, of course, its preference probability is boosted through an upgrade or a price reduction.

The possession transition probabilities from state  $\{i|j\}$  to state  $\{i|i\}$  at time  $t$  will be denoted by  $\phi_{i|ji}(t)$ . These probabilities are obtained from the relation

$$\begin{aligned}\phi_{i|ji}(t) &= S_i(t - IN_i), && j \neq i, \\ \phi_{i|i}(t) &= 1.\end{aligned}$$

The function  $S_i(\tau)$  is a subjectively estimated "penetration curve" describing the fraction of those potential customers preferring but not possessing product  $i$  who switch to it during the  $\tau$ th period after it becomes available. This penetration curve is analogous to the growth rate model of Parfitt and Collins (1968). Note that such penetration curves are assumed not to depend upon the subpopulation or the concept possessed and that they depend upon elapsed time from the introduction date,  $IN_i$ , of product  $i$ . This does not contradict suggestions in the adoption/diffusion literature implying that adoption rate is a function of the relative advantage of the innovation over the old technology since the penetration curve is applied to the pool of potential customers for the product. The relative advantage of the product over its competitor has already been accounted for in determining that potential pool, so the penetration curve must account only for factors determining how fast this potential can be realized.

Specifically, the penetration curves account for the time it takes for potential customers to become aware of the product and to develop confidence in their beliefs about its characteristics. They also account for customer delays in planning for new equipment. Finally, they account for the ability of the supplier to convert awareness and preference by a potential customer into possession. The effects of limitations on the manufacturing capacity and salesforce capacity of a supplier depend upon the aggregate (as opposed to subpopulation) potential. Dealing with this kind of limitation

is discussed in §4. In the computer printing forecasting system, we employed a simple linear function for  $s(\tau)$  which was truncated at  $S(\tau) = 1$  and had an initial jump at  $\tau = 0$ , i.e.,  $S_i(\tau) = a_i + \min[b_i\tau, (1 - a_i)]$ . This function was fit (by inspection) to subjective estimates of the time it will take the new product to achieve its maximum penetration rate and to the fractions of outstanding potential that will be captured by the various products in each month following their respective introduction dates. An alternative representation of the penetration curves  $S_i(\tau)$  which was employed in a subsequent application uses a discrete time approximation of the exponential penetration rate curve  $\hat{S}(\tau) = b \exp(a\tau)$ , truncated again at  $S(\tau) = 1$ .

It is quite common in marketing high technology products to announce a new product before it is actually available for delivery. In such cases we assumed the introduction date  $IN_i$  to be the announcement date and  $S(\tau)$  was set to zero on the initial time segment from announcement to availability. This captures the phenomenon of backorders and letters of intent which allow the creation of a potential customer pool as soon as a product is announced.

The model maintains a count of the expected number of customers in each state and updates these figures for each period by first executing the preference transitions and then the possession transitions. Let  $N_{i|j}(t)$  denote the expected population in state  $\{i|j\}$  at time  $t$ . Then  $N_{i|j}(t + 1)$  is obtained in two steps. First,  $N_{i|j}^*(t + 1) = \sum_m \phi_{mi|j}(t + 1)N_{m|j}(t)$ ; then

$$N_{i|j}(t + 1) = [1 - \phi_{i|ji}(t + 1)N_{i|j}^*(t + 1)], \quad i \neq j,$$

$$N_{i|j}(t + 1) = \sum_j \phi_{i|ji}(t + 1)N_{i|j}^*(t + 1).$$

Clearly,  $N_{i|j}(t) = 0$  for all  $t < IN_j$  and  $t < IN_i$ , i.e., prior to the introduction of products  $i$  and  $j$ . It still remains, however, to determine the initial conditions  $N_{i|j}(0)$  for all  $i$  and  $j$  that were available prior to year 0, i.e., the initial forecast year. This poses some technical difficulty since market data would only reveal the total placements of each product  $j$ , i.e.,  $W_j(0) = \sum_{i \in Z(0)} N_{i|j}(0)$ . A special procedure for determining the initial conditions  $N_{i|j}(0)$  using the aggregate placements  $W_j(0)$  and preference probabilities  $P_{i|j}(0)$  has been developed. It is based on backing up the dynamics model to the year the first product was introduced and running it forward, under simplifying assumptions, from that time to year zero, the initial year of the forecast.<sup>1</sup>

Figure 3 illustrates a set of typical product potential curves and the corresponding set of population curves produced by the procedure described in this section for four products with staggered introduction dates denoted by  $IN_1, IN_2, IN_3$  and  $IN_4$ . The potentials are calculated as described earlier by simply distributing the total market each year among the available products, in proportion to their preference probabilities free of inertia, i.e.,  $P_{i|j}^*(t)$ . This leads to a staircase behavior in which the potential of a product jumps from zero to its market share upon introduction and drops as subsequent products are introduced. This drop accounts for customers who prefer the later products but in their absence were regarded as potential for the available products. The stochastic choice model underlying the logit equation implies that such customers will exist even if the later products are inferior to the earlier ones. Thus, the drop in potential by no means indicates that later products are superior. The population curves in the lower part of Figure 3 illustrate some of the intuitively appealing properties of this model. Note the lags in the movement of the populations. Note also that products introduced earlier exceed their potential while the later ones fall short. The long-run persistence of this behavior is induced by the inertia effect captured by this model.

<sup>1</sup>The technical details of this procedure are described in an unpublished appendix which can be obtained, upon request, from the authors.

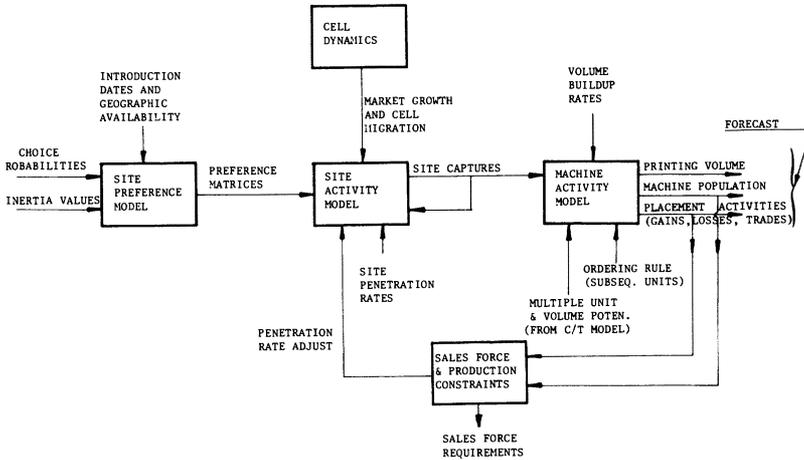


FIGURE 3. Illustrative Example of Potentials and Corresponding Population Forecast for a Four-Product Case.

The remainder of this section is devoted to demonstrating the calculation of the population curves for the first two years. We assume for this purpose that the preference probabilities with full availability,  $P_{i|j}^*(t)$ , are known in each year. In year zero since only product 1 is available we have  $P_{1|j}(0) = 1$  and  $P_{2|j}(0) = P_{3|j}(0) = P_{4|j}(0) = 0$  for  $j = 0, 1, 2, 3, 4$ . Thus, the entire market consisting of  $N$  customers prefers product 1, and assuming that they do not own any other comparable product they are placed in state  $\{1|0\}$  forming a potential customer pool for product 1. If product 1 has a first year penetration rate of  $S_1(1)$ , then by the beginning of the second year, i.e., at  $t = 1$ , the number of customers in state  $\{1|1\}$  is  $N_{1|1}(1) = S_1(1) \cdot N$ . At that time, products 2 and 3 are introduced so the preference probabilities of products 1, 2, and 3 change to

$$P_{i|j}(1) = P_{i|j}^*(1) / (P_{1|j}^*(1) + P_{2|j}^*(1) + P_{3|j}^*(1)) \quad \text{for } i = 1, 2, 3, \quad j = 0, 1, 2, 3, 4.$$

The preference probabilities of product 1 decreased by  $1 - P_{1|j}(1)$  while those of products 2 and 3 increased from zero to  $P_{2|j}(1)$  and  $P_{3|j}(1)$  respectively. These changes will cause a redistribution of the unrealized potential for product 1 and also some of the customers owning product 1 will move to states  $\{2|1\}$  and  $\{3|1\}$  and will become potential for products 2 and 3. Following the calculations described earlier, the number of customers  $\Delta N_{1|1}$  which have product 1 and will potentially trade to products 2 or 3 is given by  $\Delta N_{1|1} = -(1 - P_{1|j}(1))N_{1|1}$ . These will be distributed among products 2 and 3 in proportion to their increased preference probabilities, i.e.,

$$N_{i|1}^*(2) = \Delta N_{1|1} \cdot P_{i|1}(1) / (P_{2|1}(1) + P_{3|1}(1)), \quad i = 2, 3.$$

A similar computation can be performed for potential customers not yet captured by product 1 to obtain  $N_{1|0}^*(2)$ ,  $N_{2|0}^*(2)$  and  $N_{3|0}^*(2)$ . Products 2 and 3 will now capture, respectively,  $S_2(1)$  and  $S_3(1)$  fractions of their potential pools so their population by the end of the second year is

$$N_i(2) = N_{i|i}(2) = S_i [N_{i|0}^*(2) + N_{i|1}^*(2)], \quad i = 2, 3.$$

The population of product 1 by the end of the second year is composed of its new captures in its remaining potential pool and customers who had product 1 and were not captured by the new products. This gives

$$N_1(2) = S_1(2) \cdot N_{1|0}^*(2) + N_{1|1}(1) - S_2(1) \cdot N_{2|1}^*(2) - S_3(1) \cdot N_{3|1}^*(2).$$

The curves at the bottom of Figure 3 display the total population  $N_i(t)$  of each of the four products as function of time.

#### 4. Complications in the Computer Printing Forecasting System's Dynamics Model

There are a number of complications accounted for in the dynamics model of the computer printing forecasting system that are not captured by the basic model discussed above. Some aspects of these features are relevant only to the computer printing market, but most can be useful for other types of products. One of the more general features addresses market growth and changes in the distribution of characteristics of potential customer types.

Another problem that must be dealt with is the possible limitation on manufacturing and field support capacity, especially in the early period of product availability. The subjectively assessed penetration rates might lead to violation of such constraints and must be adjusted to reconcile that constraint.

Geographic diversity in launch strategy is yet another problem that must be dealt with. Products need not be launched at the same time in different geographical regions. However, staged launches will induce a diversity of preferences and acquisitions even among customers represented by the same market research respondent. Consequently, the expected population in each state of the model must be tracked separately for each geographical region taking into consideration the geographical distribution of different customer types.

In the computer printing market, as in other leased product markets in which usage charges are imposed, forecasts are needed for usage as well as for placements. Such usage forecasts must consider the transient effects resulting from the gradual volume build up on the newly acquired product. Furthermore, usage and placements may interact. A customer's acquisition of further units of a product may well be paced by the volume build up on units already acquired.

This section describes how the above complications can be dealt with. This is done in the context of the computer printing forecasting system for which the proposed features of the model have been implemented. Figure 4 illustrates the various components of the computer printing market dynamics model and their interactions.

##### *Growth and Change in the Sampled Universe*

Xerox planners projected that the number of computer centers of the size included in the study would increase gradually over the forecast period. They also projected changes in the mix of main frame types and in the printing volumes in the computer

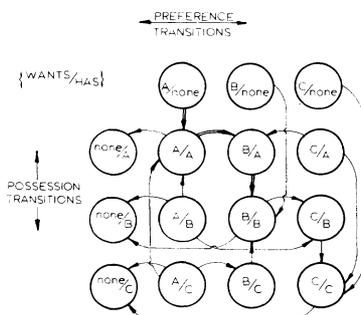


FIGURE 4. Overview of the Dynamics Model Used in the Xerox Computer Printing Forecasting System.

centers included in the forecast universe. These projections were based on extrapolation of a historical record of computer center census data published by the International Data Corporation. This census gives detailed information on all the computer installations in the U.S. The forecasts produced by the computer printing forecasting system had to account for these projected changes. This was done by constructing, based on the available data, a scenario of migrations between sampling cells and using it to modify the sampling weights associated with each respondent over time. The scenario specified the percentage of the subpopulation in a sample cell that migrated to each other sample cell each period and the number of new entries from outside any cell. Since large computer centers seldom start from scratch and typically grow out of smaller centers, projected growth in total universe size was accounted for through migrations into the cells containing the smallest computer centers. The cell migration rates were uniformly applied each period to all subpopulations in a cell modifying the expected population in each state prior to updating. Specifically, before  $N_{i|j}^k(t+1)$  was calculated,  $N_{i|j}^k(t)$  was reduced by the emigration rate for period  $t$  and then increased by its proportional share of the immigration to subpopulation  $k$ 's cell from subpopulations possessing concept  $j$ . Thus, all migrants were assumed to switch to the preferences of the subpopulations in the cells to which they migrated. However, their initially possessed product in the new cell was assumed to be the same as in their previous cell. This avoids unrealistic discontinuities in the product populations and erratic trade patterns. Immigrants from outside the cells in the universe for which we forecast were always assumed to have old technology (impact printing only) as their currently possessed product.

To express this formally, define  $c(k)$  as the sample cell into which subpopulation  $k$  is classified and  $m_{p,q}(t)$  as the rate of migration from cell  $p$  to cell  $q$  at the end of period  $t$ . (To simplify the expression, assume that there is a cell that contains the computer centers that are outside the universe for which we are forecasting and also that  $m_{p,p} = 0$  for all  $p$ .) Before other calculations for period  $t+1$  begin, we account for the migration by replacing  $N_{i|j}^k(t)$  with  $\hat{N}_{i|j}^k(t)$  given by

$$\hat{N}_{i|j}^k(t) = N_{i|j}^k(t) \left[ 1 - \sum_q m_{c(k),q} \right] + \sum_l N_{i|l}^l(t) m_{c(l),c(k)}.$$

#### *Forecasting Usage and Number of Printers*

The cost/throughput model and the data that support it give a detailed picture of how the present printing volume of each respondent would be printed if that respondent had any particular proposed nonimpact computer printing product and how many nonimpact printers he would require to print that volume. These volumes and numbers of printers were intended to reflect the thinking of computer center managers who were considering the acquisition of nonimpact printers. As such, these estimates were believed by Xerox planners to be a snapshot of what in fact would be an evolving process of nonimpact printer usage characterized by usage growth with user experience. The planners assumed that computer centers that obtained nonimpact printing would tend to introduce it cautiously. After it had proved itself, the centers would then begin to use it about as they had planned. However, after several years much of the printing that was not considered in the initial plan for obtaining nonimpact computer printers would be printed on such printers as well. Even some printing volume that, as currently printed, was technologically infeasible on nonimpact printers would eventually be replaced by printing that would be printed on them.

In order to model the evolution in the number of nonimpact printers at a computer center and the volume printed on them, we expanded the state of the basic Markov process of the dynamics model to include a record of years of experience with nonimpact printers. Volume growth curves provided by the planners (based on

subjective assessment and historical data) were used to determine the percent of potential volume transferred to nonimpact printers given the years of experience with such printers. These curves varied by type of nonimpact printer and were provided separately for three volume categories:

1. volume planned for the current nonimpact printer,
2. volume that is technologically feasible for the current nonimpact printer but not planned for transfer, and
3. volume that is currently not technologically feasible for transfer to the existing nonimpact printer at the site (e.g., card stock).

The potential volumes in each of the above categories were computed and passed on by the cost/throughput model.

To limit the state space, it was assumed that each of the volume growth curves reaches an asymptote after a finite number of years (we assumed five) so that only a limited (five-year) nonimpact printing experience record needs to be kept. To summarize the above in analytic terms, we have expanded the state space  $\{i|j\}$  denoting preferred and possessed concepts to  $\{i|j, T\}$  where  $T$  denotes number of "equivalent years" experience with nonimpact printing concept  $j$ . Preference transitions originating in state  $\{i|j, T\}$  could go to any state  $\{l|i, \tau\}$  where  $l = 1, \dots, n$  and  $\tau = \min[5, T + 1]$  if  $j \neq 1$  or  $\tau = 0$  if  $j = 1$ . It is assumed here that product 1 represents impact printers only, so while  $j = 1$  no experience with nonimpact printing is gained. When acquisition transitions take place, namely from state  $\{i|j, T\}$  to state  $\{i|i, \tau\}$ ,  $\tau$  is chosen so as to minimize discontinuity in nonimpact printing volume buildup. In other words,  $\tau$  is (to the nearest integer approximation) the number of years with product  $i$  yielding the same volume as that corresponding to  $T$  years with product  $j$ .

Based on the volume buildup, a simple heuristic was used to trigger the sequential acquisition of units of the possessed nonimpact concept. This heuristic, which was devised in an attempt to capture the planners' intuition, incorporated subjective constraints on the utilization of existing printers and the tolerated underutilization of new units as well as preference information on utilization rates at the represented site which was computed and passed on by the cost/throughput model.

#### *Manufacturing and Salesforce Constraints*

It was mentioned earlier that penetration rates, which drive the possession transitions, reflect customer and marketing restrictions on the realization of preferences. These rates, however, are also used to accommodate manufacturing and salesforce constraints. This is done through a feedback loop in which the new units placed (i.e., newly leased) in each year are totaled for the entire market. Similarly the number of sales representative hours needed to handle the new and existing placements are computed in a separate sales model using estimates based upon historical salesforce productivity statistics. These figures are compared to exogenously specified production and salesforce constraints. If any of these constraints is exceeded, the penetration rates of the corresponding products are reduced and the calculations repeated. Fortunately, the new placement and salesforce needs are nearly linear in the penetration rates (with small deviations due to integer approximations). Thus, the needed modification to the penetration rates can be predicted quite accurately and the constraints are typically satisfied after one iteration. Specifically, we can use the fact that with a zero penetration rate there would be no new placements of the particular product. Hence, we can calculate the percentage reduction in new unit placements needed and achieve that reduction through the same percentage decrease in the penetration rate. If constraints are violated simultaneously on two competing products, the procedure is similar but more than one iteration may be needed to satisfy all the binding constraints.

### *Geographic Marketing Diversity*

Not all computer printers that are available are available everywhere in the country. Some suppliers may make their product available anywhere nationally at the time of its introduction. Others may ultimately make theirs available nationally, but only after a long period of gradually relaxing geographical restrictions on marketing. Still others may permanently restrict their marketing to a relatively few large metropolitan areas. In order to avoid the aggregation errors inherent in having a single product availability scenario, a geographically disaggregated version of the dynamics model was created. With the exception of the manufacturing capacity constraints which were applied nationally, this model is just a series of replications of the basic dynamics model. Each replication has its own product availability scenario. This approach, while general and flexible, essentially multiplies the computer time for running the dynamics model by the number of geographic regions. This had consequences for the use of this feature that are discussed below in §5.

In using this capability, we defined the geographic areas by size of metropolitan area rather than as contiguous areas. Since the market research sample, while geographically diverse, was not geographically representative, we chose to ignore the actual geographic location of the respondents. Instead we allocated each respondent's sample weight to subpopulations in each geographical area in proportion to the number of computer centers (of the size studied) in the area.

## **5. The Use of the Forecasting System and of the Dynamics Model**

The paper by Oren, Rothkopf and Smallwood (1980) describes in detail the use by Xerox of the computer printing forecasting system. In this section, we review that description and elaborate on the use of the dynamics model. We precede these descriptions with a discussion of the computer implementation of the forecasting system; we follow them with a discussion of some important characteristics of the forecasting system and the dynamics model for the uses described.

The computer printing forecasting system was programmed in APL and run on a Sigma 9 computer in either timesharing or batch mode. It is organized into two large programs and several smaller ones. The first large program contains the multiperiod, multirespondent value model. This program uses input files of respondent data and planning assumptions. When used with a typical set of major products and run for 14 years (enough for initialization and a forecast that covers 10 years from the proposed product's introduction), this program requires about 3 hours of Sigma 9 CPU time. It creates a large file containing each respondent's conditional values, nominal volume, and preferred configuration for each printer in each period. A small program containing the choice model uses this file along with sample weights and a product availability scenario to calculate gross potentials for each product in each year. The second large program contains the choice model and the dynamics model. It also uses the file created by the value model to create a large output file containing a completely disaggregate forecast. Another small program is run on this output file to summarize it and create aggregated forecasts.

The computer printing forecasting system was completed in the middle of 1976. It was created and initially used to support decision making with respect to the completion of development of a controversial proposed new high speed computer printer (now known as the Xerox 9700). After the computer code and market research data were debugged and the model calibration completed, the model was used in a familiarization mode to explore various competitive scenarios and the sensitivities of the model to the many planning assumptions used in it. The credibility of the model

necessitated that the cause of any surprising result be identified. This tracing process often led to modification of planning assumptions or a revision of the planners' intuitive understanding of the market. After the frequency of surprises diminished greatly, the model users presented the model, the market research, and their forecast and market analysis to top corporate management and its staff. After close examination of the model and the market research and after the analysis was expanded using the forecasting system to analyze the effects of possible new technological threat envisioned by corporate staff, corporate staff reversed its previous position, and management approved the completion of development and launch of the proposed printer.

Subsequently the system was used to examine the market effects of various tactical alternatives such as pricing, product announcement timing, and geographically constrained launch strategies. More often than not, such analysis simply confirmed management's prior judgement. However, experimentation with the model did produce an unusual combination of pricing plans that seemed advantageous and was adopted. The system was also used to evaluate the market impact of proposed product enhancements and of proposed new products such as the medium speedscanning laser computer printer announced by Xerox in 1980.

For several years after its completion, the forecasting system was used with updated planning assumptions as part of Xerox's Printing Systems Division's long-range planning process. The model, when supplied with realistic planning assumptions, has tracked actual developments in the computer printing market with a precision well within the range that could be expected from the size of the market research sample. Some additional details on the use of the forecasting system are contained in the Oren, Rothkopf and Smallwood (1980) paper.

During the initial familiarization phase of model operation, the dynamics model was used regularly on the results of each run of the value/choice model after these results were inspected to make sure that they were reasonable. These initial runs were made using seven geographic regions. These runs took about 3 hours of CPU time on a Sigma 9 computer. Typically, they were made overnight. All forecasts presented to top management were prepared using such runs.

After some experience with the system, the product planners developed the ability to estimate fairly accurately the effect on a forecast of a given change in potentials. Furthermore, some dynamics model runs were made with the whole country treated as a single geographic region. Such runs were cheaper and faster and were sometimes used to estimate independently the results of a full run with seven regions.

For each important run of the value model, several runs with the choice/dynamics model were usually made to examine the effects of alternative product availability scenarios. A few dynamics model runs were made to examine marketing strategy issues. In particular, runs were made with modified penetration curves as described above to examine the possible effects of timing in the announcement of new printers. Runs were also made for planning purposes to examine the effects of geographic launch strategy; the actual decisions on widening the territory of the initial launch, however, were made adaptively in response to market experience.

## 6. Model Evaluation

One of the most common questions concerning the merit of the forecasting system is: "How accurate is the forecast?" For several years after its completion the system's forecast was accurate to a degree well within the tolerance associated with sampling error. In 1980 Robert V. Adams, President of Xerox's Printing System Division, stated "since its completion in 1976, the forecasting system has tracked the market extremely

well.<sup>2</sup> It is also significant to note that the system has been particularly accurate in forecasting competitive placements. These, unlike Xerox's own placements, could not have been influenced by self-fulfillment of prophecies based on the forecast.

While accuracy of the forecast is highly desirable, it is only an ex-post measure of merit, which is not available when the decision to use the model is being made. Furthermore, one must realize that a comparison of sales forecast with actual sales history would primarily measure the accuracy of the market research data and the subjective inputs reflecting the planner's beliefs at the time the forecast was created. For product planning systems of this kind an important measure of merit is their credibility at the time of use.

As evident from the above description, the forecasting system was used primarily as a tool for product planning and market analysis through extrapolation of various assumptions and examination of their implications. To a large extent, this system, like other forecasting systems of this type, was used as a decision aid for developing a credible business plan that is consistent with a set of plausible assumptions. This process took place in a corporate framework in which business plans are tested in an advocacy system. Thus, the forecasting system has been used, at least initially, as a tool for developing a market scenario and a set of plausible assumptions to support the proposed product strategy. The same model was also used by the corporate reviewers to scrutinize that strategy. The consensus forecast was a result of negotiation about the proper set of assumptions and subjective inputs that should be used. The fact that both sides, the product planners and corporate reviewers, were willing to accept the model as a medium for their debate and use it for evaluating the implications of their assumptions is clear evidence of its credibility.

A number of factors tend to influence the credibility of a model. A vital contributor to credibility is model transparency. The causes of any result, especially one that seems anomalous, must be traceable in simple intuitive steps back through intermediate results to the data or assumption that gave rise to it. Important decisions about major new products are not made by a model but by executives who may have been influenced by a model. It is our experience that in order for a model to change an executive's opinion, it is necessary for such detailed analyses to be made. Of course, such comparisons don't always change opinions. Sometimes they lead to improvements in the model and sometimes, because of the difficulty of improving the model, they lead to allowances for recognized, but limited, deficiencies in the model.

A related contributor to model credibility is the ability to compare and reconcile assumptions and intermediate results with those of other forecasts. This implies, for example, that there are advantages to the use of a familiar penetration curve concept rather than a completely new one and that, if a new one is used, it should be carefully compared to the prior method under a variety of assumptions. In general, the more points of comparison the better. The ease of use of a model contributes to the ability to make comparisons and, thus, to the ability to establish credibility in a timely manner.

Finally, the *process* of model creation can play a vital role in establishing a model's credibility. In particular, the value of involving the user and his staff in model design is immense. This involvement during the creation of the computer printing forecasting system is described in Oren, Rothkopf and Smallwood (1980). Such involvement helps to assure that the right problem and all of its important aspects are addressed by the model. It helps to improve the design tradeoffs involved in creating the model. It helps to insure that the model user will understand the limitations of the model and that he will be confident that he does understand them. Finally, it largely eliminates both the rational and irrational aspects of the "not invented here" syndrome.

<sup>2</sup>For the complete statement, see Oren, Rothkopf and Smallwood (1980).

Several aspects of the market dynamics model improved its credibility relative to the prior process of converting potentials into forecasts. Its assumptions were simpler and more systematic. Yet they were similar to assumptions made before. Thus, they could be compared allowing judgement based on experience with prior forecasts to be applied. The simplicity of the assumptions combined with the relative speed of calculation allowed assumptions to be varied so that their reasonableness could be examined. The simplicity of the assumptions combined with the disaggregated nature of the whole forecasting system contributed to transparency; when a forecast changed, it was easy to trace the chain of causation of the change and examine each link for reasonableness. Finally, the person responsible for the prior manual calculations was part of the working group that created the forecasting system. His participation contributed to both the actual and the perceived reasonableness of the design assumptions of the market dynamics model.

## 7. Discussion

Many aspects of the dynamics model in the computer printing forecasting system are identical or at least similar to calculations that would be useful in new product strategy models designed for many other markets. We believe that similar dynamics models will be particularly helpful in studies of markets in which there is a substantial delay between customer preference change and product acquisition. Markets involving capital equipment and components of complex systems tend to have such characteristics. Similar dynamics models may also be helpful in studies of markets in which marketing strength, as opposed to product characteristics, plays an important role. Models of both kinds of markets would frequently benefit from calculations that account for both supplier and customer induced delays in converting potential product preferences into product deliveries. Such models may also benefit from calculations that attempt to account for delays between the partial adoption of a new product to its full adoption, for the effect of experience with a product on future product preferences, and for changes in market composition. Such models, if they are at all disaggregated, will require initialization calculations.

However, some aspects of the dynamics model in the computer printing forecasting system are particularized to the characteristics of the market for high speed nonimpact computer printers. For example, most new product strategy models will not need to predict product usage. More importantly, some of the particulars—especially omissions—may not be suitable for other market dynamics models. For example, the dynamics model in the computer printing forecasting system does not allow any possession transitions to second and lower choice products even if these would be preferred to the present product. It also assumes that each member of the universe of potential customers satisfies his own needs at any given time with only one product concept. Mixtures of products are not allowed. (A more recent new product strategy model in Xerox has relaxed this assumption.)

The penetration curves that play such a fundamental role in the computer printing forecasting system's market dynamics model are taken as subjective input. There may well be situations in which it would be desirable to attempt to model them explicitly in terms of marketing policy variables or, perhaps, to estimate them formally from historical data.

There are several characteristics of the computer printing forecasting system and its market dynamics model, especially their extreme degree of disaggregation, that are appropriate because of the nature of the system's market research base. For models in which the input to the value model is largely subjective, a much greater degree of aggregation may be appropriate, and the resulting decrease in the computational load

might make the inclusion of other complications, such as feedback from the dynamics model to the value model, more attractive modeling alternatives.

**Appendix A. Derivation of Transition Probabilities for the Basic Model**

For convenience, we shall employ the shorthand notation  $\{A\}$  to denote probability of event  $A$ ,  $\{A | B\}$  to denote probability of  $A$  given  $B$  and  $\{A, B\}$  to denote the joint probability for  $A$  and  $B$ . We also use the variables  $s$  and  $s^1$  to represent the respective alternatives preferred in period  $t$  and  $t + 1$  by customers having a particular product  $j$ . Thus, in terms of our earlier notation we have

$$P_{i|j}(t) = \{s = i\}, \quad P_{i|j}(t + 1) = \{s^1 = i\} \quad \text{and} \quad \phi_{ik|j}(t + 1) = \{s^1 = k | s = i\}.$$

For further notational simplicity, we will suppress the subscript  $j$  denoting the currently owned product.

**THEOREM.** *Let*

1.  $\{s = i, s^1 = i\} = \min\{P_i(t), P_i(t + 1)\}$  and
2.  $\{s = i, s^1 = k | s \neq s^1\} = \{s = i | s \neq s^1\} \{s^1 = k | s \neq s^1\}$

(the interpretation of these conditions is given in the main text). Then

$$\begin{aligned} \phi_{ik|j} &= \min\{P_i(t), P_i(t + 1)\} / P_i(t) \quad \text{for } i = k, \\ &= \frac{2(\max\{0, (P_k(t + 1) - P_k(t))\})(\max\{0, (P_i(t) - P_i(t + 1))\})}{P_i \sum_{k=1}^n |P_k(t + 1) - P_k(t)|} \end{aligned}$$

**PROOF.** For  $i = j$ ,

$$\{s^1 = i | s = i\} = \{s = i, s^1 = i\} / \{s = i\} = \min\{P_i(t + 1), P_i(t)\} / P_i(t). \tag{A.1}$$

For  $j \neq i$ , assumption 2 implies

$$\begin{aligned} \{s^1 = k | s = i\} &= \{s^1 = k, s^1 \neq s | s = i\} = \{s^1 = k, s = i | s^1 \neq s\} \{s^1 \neq s\} / \{s = i\} \\ &= \{s^1 = k, s^1 \neq s\} \{s = i | s^1 \neq s\} \{s^1 \neq s\} / \{s = i\}. \end{aligned} \tag{A.2}$$

Using Bayes rule, we then have

$$\begin{aligned} \{s^1 = k | s = i\} &= \{s^1 = k, s^1 \neq s\} \{s^1 \neq s | s = i\} / \{s^1 \neq s\} \\ &= \{s^1 = k, s \neq k\} \{s^1 \neq i | s = i\} / \{s^1 \neq s\}, \end{aligned} \tag{A.3}$$

but

$$\begin{aligned} \{s^1 = i | s = i\} &= 1 - \min\{P_i(t + 1), P_i(t)\} / P_i(t) \\ &= \max\{0, (P_i(t) - P_i(t + 1)) / P_i(t)\}, \end{aligned} \tag{A.4}$$

$$\{s^1 \neq k | s = k\} = \{s = k\} - \{s = k, s^1 = k\} = \max\{0, (P_k(t) - P_k(t + 1))\}. \tag{A.5}$$

Also,

$$\{s^1 \neq s\} = \sum_{k=1}^m \{s^1 \neq k, s = k\} = \sum_{k=1}^m \{s^1 = k, s \neq k\}. \tag{A.6}$$

From (A.4), (A.5) and (A.6),

$$\begin{aligned} &\sum_{k=1}^n (\max\{0, (P_k(t) - P_k(t + 1))\} + \max\{0, (P_k(t + 1) - P_k(t))\}) \\ &= \sum_{k=1}^m |P_k(t + 1) - P_k(t)| = 2\{s^1 \neq s\}. \end{aligned} \tag{A.7}$$

The required result follows by substituting (A.4), (A.5) and (A.7) into (A.3).<sup>3</sup>

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<sup>3</sup>This paper was received February 1983 and has been with the authors for 3 revisions.

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